MAPPING THE HEDONIC TREADMILL: A DYNAMIC MODEL OF EMOTIONAL VALENCE

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1. Introduction

Humans maintain countless systems in homeostasis through complex feedback loops, and emotional regulation is no exception. By all accounts, it should be easy to study emotion; we as humans think about, discuss, and modulate our emotions on a near-constant basis. And yet, evidence suggests that measuring an individual's emotional state is one of the most difficult problems in psychology.

Though there is far from a consensus, many psychologists characterize emotional response using three dimensions: valence, arousal, and approachavoidance [Mau09]. This paper will focus solely on valence. At its most basic, valence contrasts positive and negative emotional states ranging from anger and sadness to ambivalence to happiness and joy. Valence is a core aspect of the human emotional landscape, and essential in the valuation process [Bar05]. Valence, as a measure of mood and satisfaction, is also one of the easiest dimensions for individuals to assess and communicate.

In this paper, we will propose a general dynamic model for studying valence regulation and response to stimulus. The model takes valence state and hedonic setpoint as dependent variables and incoming emotional stimulus as an independent variable. Though valence is a qualitative measure, we will represent it here as a real number. The magnitude of this value is not significant; we only consider larger values to have greater positive valence, and smaller values to have greater negative valence. A value of zero is considered an emotionally-ambivalent state — one in which an individual feels neither positively nor negatively.

At the core foundation of the model is the notion of the hedonic treadmill: the tendency for individuals to quickly return to a baseline emotional state despite major events or life changes. This baseline state is referred to as the hedonic setpoint, and its effects are readily-felt: barring major trauma, individuals tend to recover from negative events and get over positive ones.

While previous research held that the hedonic setpoint is more-or-less fixed from birth, more recent research [Die06] shows that is is in fact variable and can shift up and down over the course of one's life. It is this behaviour that we hope to understand using the proposed model.

1.1. Emotional Processing Axioms.

In order to effectively formulate a mathematical model of valence dynamics, we have distilled a number of psychological principles from existing literature into a set of axioms on emotional processing. These axioms served as guiding principles while developing the model, and will be referred back to while discussing our findings.

- **A1** Affect - Emotional impulses result in a change of perceived valence state.
- **A2** Adaptation - The current valence state slowly tends to a baseline state (the hedonic setpoint) over time.
- **A3** Drift - The hedonic setpoint can change in response to repeated stimulus over a long period of time. Large events (particularly traumatic) tend to exaggerate this effect.
- **A4** Contrast - Emotional impulses are perceived with different magnitudes depending on current valence state.
- **A5** Habituation/Desensitization - Repeated emotional impulses have diminishing returns. Constant stimulus results in a steady valence state.

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2. Dynamic Model

We propose a second-order dynamic model of emotional valence based on the above axioms:

$$\begin{cases} \dot{x} + \tau_x^{-1} x - \tau_x^{-1} y = p\left(E(t), x, y\right) \\ \dot{y} - \tau_y^{-1} x + \tau_y^{-1} y = 0 \end{cases} \tag{1}$$

where $x, y \in \mathbb{R}$ represent the valence state and hedonic setpoint respectively, $E \colon \mathbb{R}^+ \mapsto \mathbb{R}$ the absolute emotional stimulus experienced at a given time, and $p \colon \mathbb{R}^3 \mapsto \mathbb{R}$ the perceived valence of said stimulus. $\tau_x, \tau_y \in \mathbb{R}^+$ are time constants governing the rates of adaptation and setpoint drift, respectively; it is assumed that $\tau_y > \tau_x$.

The term $\left(\tau_x^{-1} x - \tau_x^{-1} y\right)$ drives the current valence state x toward the baseline y (Axiom **A2**), while the term $\left(-\tau_y^{-1} x + \tau_y^{-1} y\right)$ causes the baseline to drift toward the current state (Axiom **A3**). These terms were modelled after Newton's law of cooling as applied to two coupled masses of different thermal coefficients.

The inhomogeneous part of $\dot{x}$, $p\left(E(t), x, y\right)$, corresponds to Axiom **A1**: experienced emotional stimuli, transformed by a perception function, have a direct effect on the valence state x. Conversely, $\dot{y}$ is homogeneous; the hedonic setpoint can only be changed indirectly via the behaviour of x.

Axioms **A4** and **A5** are accounted for in the formulation of p, discussed in detail in Section 3.

2.1. Convergence.

In order to demonstrate convergence of x and y as required by Axiom **A2**, we examine the homogeneous case where the perceived stimulus $p\left(E(t), x, y\right) = 0$ after some time t_0[1]. It is plain to see that under the absence of external input, the system in Equation 1 is at equilibrium when $x = y$. In fact, under these conditions, the diagonal is a stable attractor, and all initial values x_0 and y_0 will eventually converge.

We seek to show that x and y exponentially approach the same value in an absence of external stimulus. Let $\Delta = x - y$ represent the deviation of current valence state from the hedonic setpoint. Then $\dot{\Delta} = \dot{x} - \dot{y} = -\left(\tau_x^{-1} + \tau_y^{-1}\right)\Delta$, which implies $\Delta(t) = \Delta(t_0)\exp\left(-\frac{t-t_0}{\tau_\Delta}\right)$, where, for convenience, $\tau_\Delta = \left(\tau_x^{-1} + \tau_y^{-1}\right)^{-1}$ represents the time constant of equilibrium convergence. Thus,

$$\exists\, t_0 \in \mathbb{R}^+ \mid p\left(E(t), x, y\right) = 0 \text{ for } t > t_0 \implies \lim_{t\to\infty} \Delta(t) = 0 \iff \lim_{t\to\infty} x(t) = \lim_{t\to\infty} y(t) \tag{2}$$

demonstrating convergence of x and y as required by Axiom **A2**.

2.2. Calculating Drift.

In order to determine the change in hedonic setpoint $\lim_{t\to\infty} y(t) - y_0$ induced by a given input, we will first find the impulse response of the system. When $p\left(E(t), x, y\right) = 0$, Equation 1 can be formulated as the linear system

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \mathbf{A} \begin{bmatrix} x \\ y \end{bmatrix} \text{ where } \mathbf{A} = \begin{pmatrix} -\tau_x^{-1} & \tau_x^{-1} \\ \tau_y^{-1} & -\tau_y^{-1} \end{pmatrix}$$

has eigenvalues $\lambda_1 = 0, \lambda_2 = -\tau_\Delta^{-1}$ corresponding to eigenvectors $\vec{v_1} = \left[\begin{smallmatrix} 1 \\ 1 \end{smallmatrix}\right], \vec{v_2} = \left[\begin{smallmatrix} -\tau_x^{-1} \\ \tau_y^{-1} \end{smallmatrix}\right] = \left[\begin{smallmatrix} -\tau_y \\ \tau_x \end{smallmatrix}\right]$, respectively. Using linear analysis, we find that a trajectory beginning at equilibrium at a point (y_0, y_0) and subject to a *perceived* valence-impulse of magnitude m at time t_0 — that is, one for which $p\left(E(t), x, y\right) = m\delta(t - t_0)$ — results in the analytic solution

$$\begin{cases} x(t) = y_0 + m\dfrac{\tau_\Delta}{\tau_y} + m\dfrac{\tau_\Delta}{\tau_x}\exp\left(-\dfrac{t-t_0}{\tau_\Delta}\right) \\ y(t) = y_0 + m\dfrac{\tau_\Delta}{\tau_y} - m\dfrac{\tau_\Delta}{\tau_y}\exp\left(-\dfrac{t-t_0}{\tau_\Delta}\right) \end{cases} \text{for } t \geq t_0 \tag{3}$$

from which is is clear to see that $x(t) = (y_0 + m)$ at time t_0 and $\lim_{t\to\infty} x(t) = \lim_{t\to\infty} y(t) = y_0 + m\frac{\tau_\Delta}{\tau_y}$; thus, the system regains equilibrium with time constant τ_Δ and the setpoint shifts by a linear factor $m\frac{\tau_\Delta}{\tau_y}$. This is consistent with Axiom **A3**: the setpoint drift is directly proportional to the perceived magnitude of the driving stimulus.

[1] This is quite a reasonable restriction; to the author's knowledge, there have been no instances of emotional perception post-death.

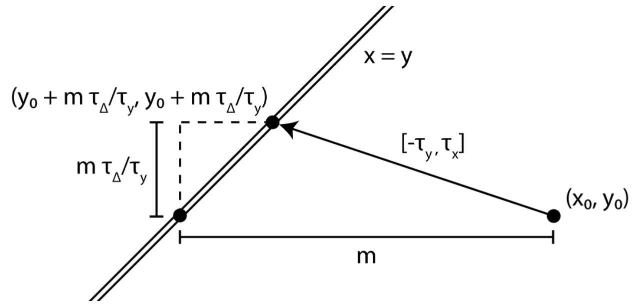

FIGURE 1. Geometric proof of setpoint drift calculation

A geometric proof of this is perhaps more illustrative: as trajectories of horizontal distance m from the line $x = y$ approach equilibrium along $\vec{v_2}$, they intersect the diagonal at point $x = y = y_0 + m\frac{\tau_\Delta}{\tau_y}$ as seen in Figure 1.

This result is significant: because perceptual impulses are time-invariant and linear with respect to behavior at infinity, we may generalize the impulse response behaviour to arbitrary stimuli. For systems starting at equilibrium at the point (y_0, y_0),

$$\lim_{t\to\infty} x(t) = \lim_{t\to\infty} y(t) = y_0 + \frac{\tau_\Delta}{\tau_y} \lim_{t\to\infty} \int_0^t p\left(E(t), x(t), y(t)\right)\, dt \tag{4}$$

provided that $\exists\, t_0 \in \mathbb{R}^+ \mid t > t_0 \implies p\left(E(t), x(t), y(t)\right) = 0$ as before.

At risk of interpreting the above too literally, Equation 4 shows an interesting result: one's day-to-day valence baseline is directly proportional to the sum of their perception of life events.

3. Perception Function

As explained in Section 2, the p function transforms an absolute emotional stimulus E to a perceived valence affect. In order to comply with Axioms **A4** and **A5**, a suitably-nonlinear perception function had to be devised such that the effect of repeated stimuli would be reduced, and constant stimulus would result in a steady state solution. In order to meet these goals, the following restrictions were imposed on p:

- Bias-free: $\forall x, y \in \mathbb{R}, E(t) = 0 \implies p\left(E(t), x, y\right) = 0$
- Sign-preserving: $\forall x, y \in \mathbb{R}, |E(t)| > 0 \iff |p\left(E(t), x, y\right)| > 0$
- Axiom **A4**: $\forall c \in \mathbb{R}, y_1 > y_2 \implies |p\left(c, y_1, y_1\right)| < |p\left(c, y_2, y_2\right)|$
- Axiom **A5**: $\forall c \in \mathbb{R}, E(t) \equiv c \implies \lim_{t\to\infty} x(t), y(t)$ exist
- Axiom **A5**: $\forall c \in \mathbb{R}, x \neq y \implies |p\left(c, x, y\right)| < |p\left(c, y, y\right)|$

After substantial experimentation, we found that $p\left(E(t), x, y\right) = E(t) \exp\left(-k_1(x-y)^2 - k_2 y^2\right)$ was a satisfactory formulation (Figure 2). This perception function weights incoming stimuli based on the current valence deviation from the hedonic setpoint (controlled by $k_1 \in \mathbb{R}^+$, dampening the effects of continued/repeated stimulus) and the value of the setpoint (controlled by $k_2 \in \mathbb{R}^+$, preventing boundless growth). The constants k_1 and k_2 effectively parametrize how well one copes with stress and change. These constants can change over time as one learns to manage emotions in different ways, but are considered fixed for the purpose of analysis.

This formulation for p is certainly not accurate. A complete model would be intractably complex, and must necessarily factor in the personality, emotional history, and coping methods of the individual. Nevertheless, by treating $E(t)$ as a normalized input and applying state-based weighting, we can get a general overview of the model's dynamics.

4. Trajectories of Selected Stimuli

The following are some sample trajectories of selected input stimuli along with light commentary.

N.B. Unless otherwise noted, sample trajectories use values $\tau_x = \frac{1}{7}$, $\tau_y = 1$, $\tau_\Delta = \frac{7}{8}$, $k_1 = 10$, and $k_2 = 2$.

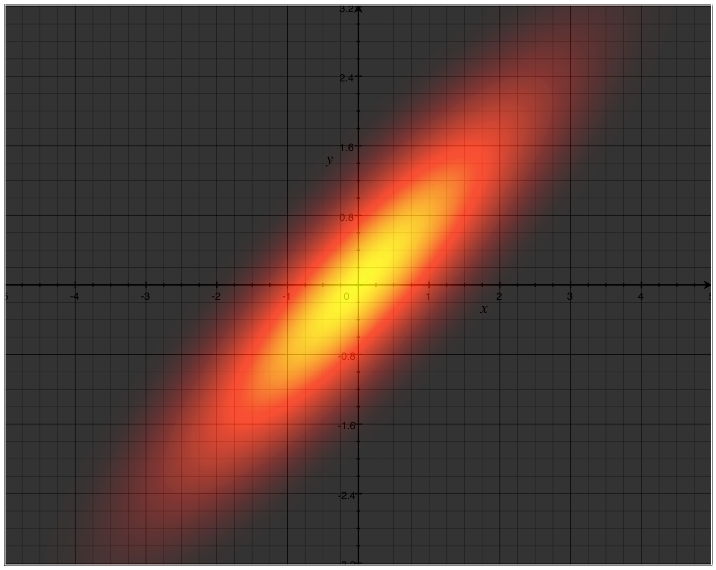

FIGURE 2. Plot of $p\left(1,x,y\right)$ for $k_1 = 1.0, k_2 = 0.3$

4.1. **Weighted Impulse Response.** The impulse response of the system to a valence-input at time t_0 is easily derived from the results of Section 2.2: for an initial condition (not necessarily at equilibrium) at (x_0, y_0) subject to an impulse $E(t) = m\delta(t - t_0)$, the system tends to $\lim_{t\to\infty} x(t) = \lim_{t\to\infty} y(t) = y_0 + m\frac{\tau_\Delta}{\tau_y}\exp\left(-k_1(x_0 - y_0)^2 - k_2 y_0^2\right)$.

As before, individual events prompting an emotional response result in a short-term deviation of current valence state from baseline coupled with a small shift in hedonic setpoint. However, the addition of the p-function means that the magnitude of the resulting shift is dependent on the pre-impulse setpoint: the farther the setpoint from ambivalence, the lower the magnitude of the shift, largely controlled by the k_2 term. Note the dependence of baseline shift on initial condition in Figure 3.

4.2. **Weighted Impulse Train.** Suppose $E(t)$ takes the form of an impulse-train: a series of N regularly-spaced valence-impulses of variable magnitude.

$$E(t) = \sum_{n=0}^{N} m_n \delta(t - n\Delta t)$$

Calculating $\lim_{t\to\infty} y(t)$ is then a matter of setting up a piecewise recurrence relationship using the same form as the above weighted impulse response. In cases where $\Delta t \gg \tau_\Delta$, this is simple; the valence state has a chance to reach equilibrium between impulses, and each impulse shifts the baseline by proportionally smaller amounts (Figure 4).

When Δt is relatively small, k_1 has a greater effect: if the model has not had time to reach equilibrium between subsequent impulses, latter stimuli will be strongly diminished as the individual becomes emotionally-saturated (Figure 5).

4.3. **Cyclic Stimulus.** Due to the effect of the p function, even unbiased cyclic inputs can result in net positive or negative setpoint shifts; note the gradual trajectory drift in Figure 6.

5. Conclusion

While the model in Equation 1 shows promise, there are nevertheless some shortcomings. A possible area for improvement would be the use of different multiplicative factors depending on the signs of $E(t)$ and $E'(t)$, incorporating effects such as positive/negative biases in perception and further refining the model's representation of Axiom **A5**. Existing literature suggests that factors such as an individual's Neuroticism level and past experiences result in differing sensitivity toward stimuli with positive and

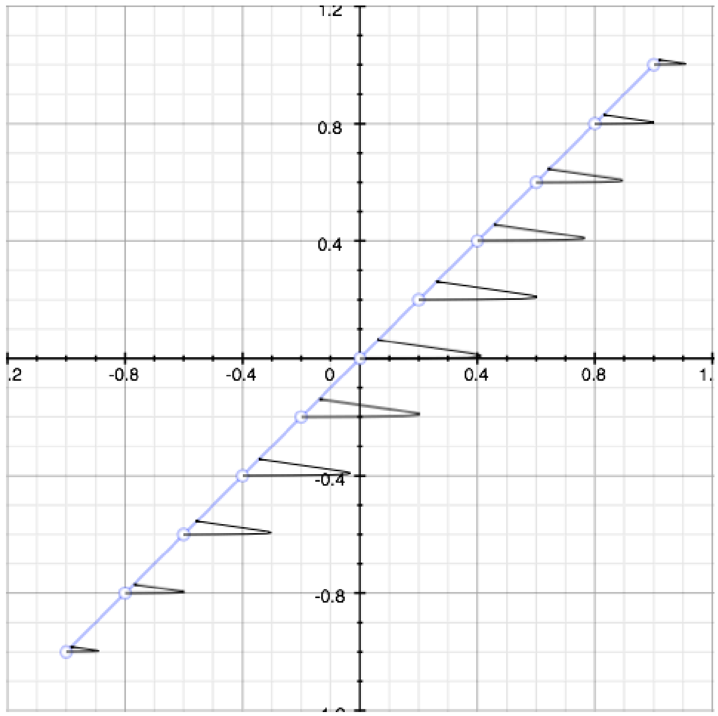

Figure 3. Impulse response from various equilibrium initial conditions

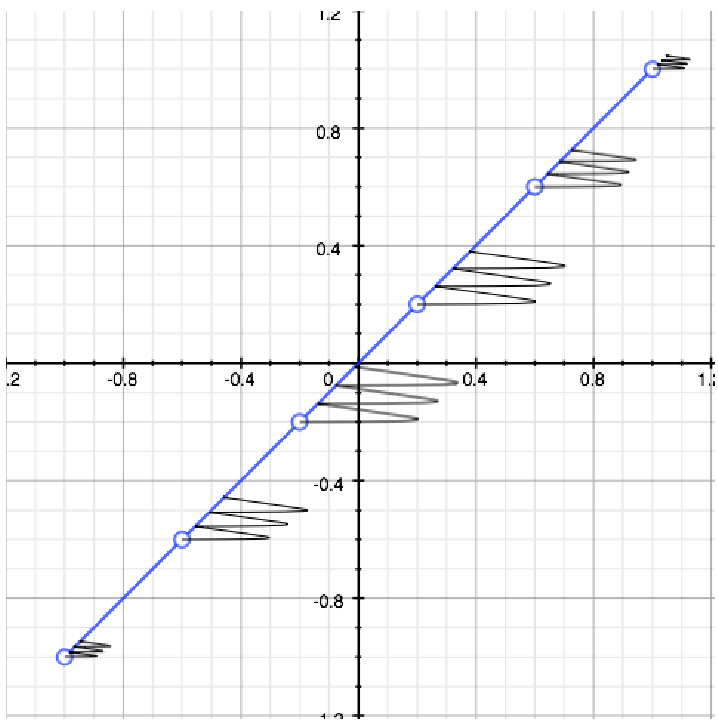

Figure 4. Impulse train with wide pulses

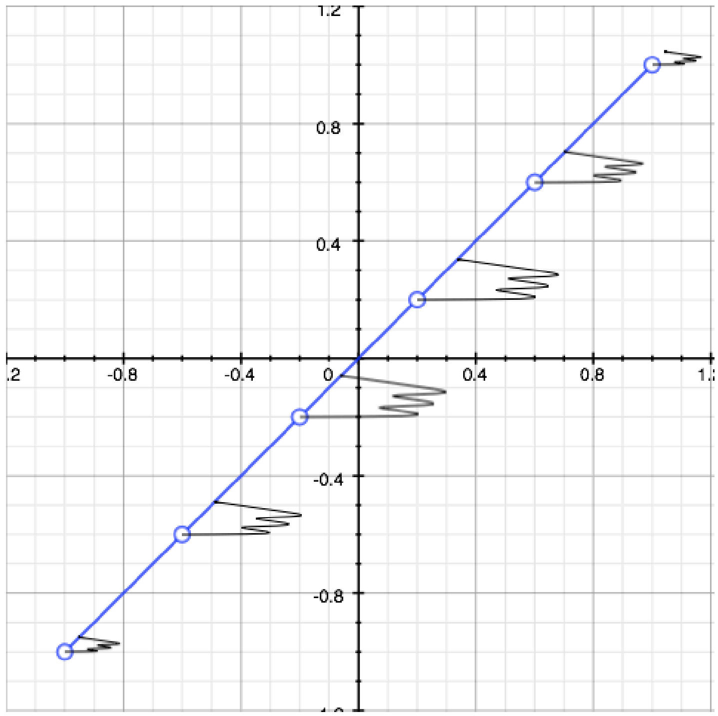

FIGURE 5. Impulse train with narrow pulses

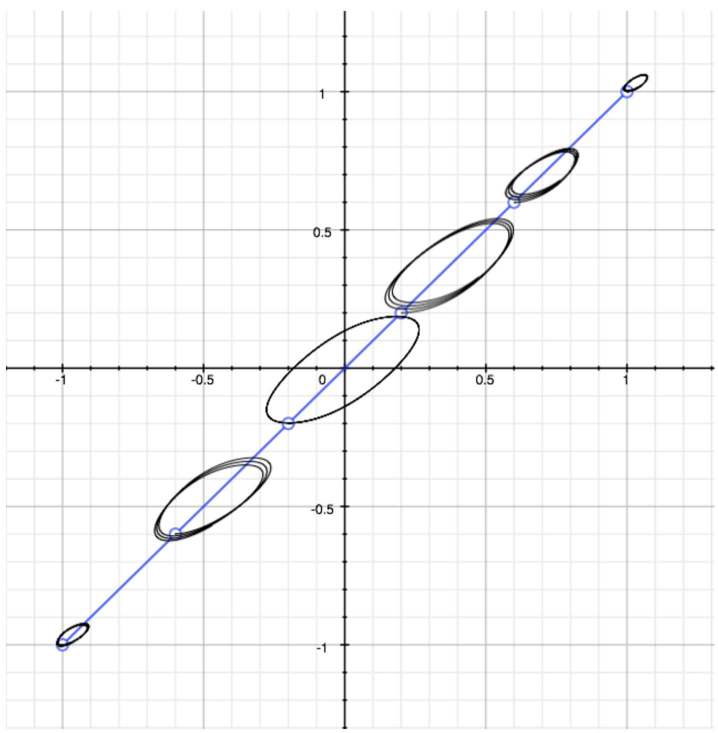

FIGURE 6. Unbiased periodic input

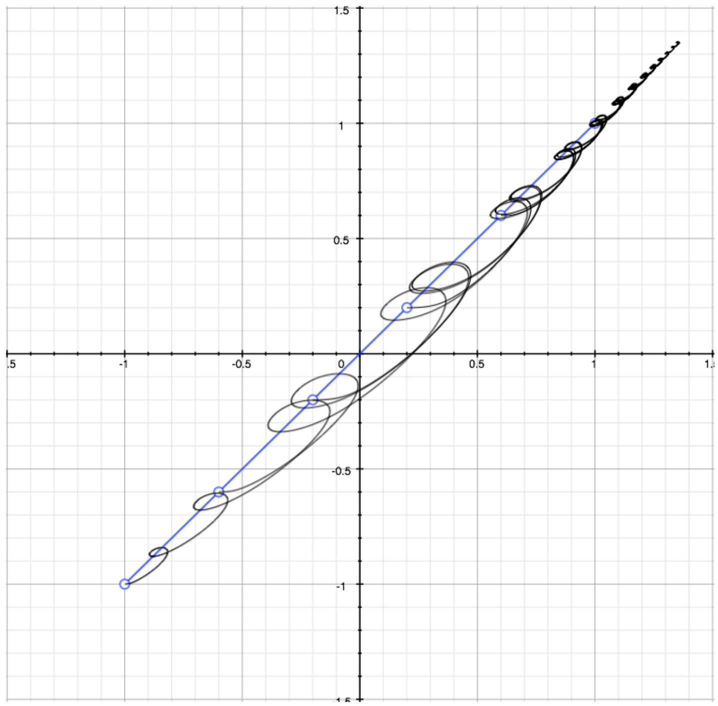

FIGURE 7. Biased periodic input

negative valence; a possible intermediate function is $S_{a,b}(E(t)) = a\ln\left(\frac{e^{E(t)}+1}{2}\right) - b\ln\left(\frac{e^{-E(t)}+1}{2}\right)$ such that S is continuous, $\lim_{E(t)\to\infty} S'_{a,b}(E(t)) = a$, $\lim_{E(t)\to-\infty} S'_{a,b}(E(t)) = b$, $S_{a,b}(0) = 0$, and $S'_{a,b}(0) = \frac{a+b}{2}$. It is possible that even a factor $S(E(t), y) = e^y \ln\left(e^{E(t)} + 1\right) - e^{-y}\ln\left(e^{-E(t)} + 1\right)$ could play a part in assessing valence perception biases.

A more accurate model should also take the neurotransmitters responsible for emotional regulation — serotonin, noradrenaline, and dopamine — into account as additional dimensions in order to model the effects that their release, uptake, saturation, and depletion have on emotional states. This would allow a model to include behavior such as the setpoint overshoot that occurs after stimulus of large magnitude.

Perhaps seeking an all-encompassing model for human emotion is a hopeless task, dependent on too many variables to quantify. Still, the analysis afforded by even this simple model is helpful in gaining intuition behind the feedback loops that maintain emotional homeostasis.

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